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Solve the equation : $1 + 5 + 9 + 13 + \dots + x = 1326$

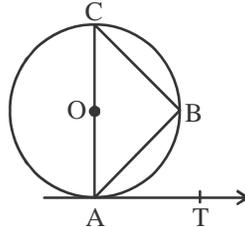
□□□□ $\frac{n}{2}(1 + x) = 1326$... (i)

$x = 1 + (n - 1) \times 4$... (ii)

Solving (i) and (ii) $x = 101$

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In Fig. 4 AB is a chord of circle with centre O, AOC is diameter and AT is tangent at A. Prove that $\angle BAT = \angle ACB$.



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□□□□ $\angle BAC = 90^\circ - \angle BAT$... (i)

In $\triangle BAC$, $\angle B = 90^\circ$

$\therefore \angle BCA = 90^\circ - \angle BAC$

or $\angle ACB = \angle BAT$ (Using (i))

3□

If $\tan \theta = \frac{3}{4}$, find the value of $\left(\frac{1 - \cos^2 \theta}{1 + \cos^2 \theta}\right)$

□□□□ $\sec^2 \theta = 1 + \frac{9}{16} = \frac{25}{16}$

$\therefore \cos^2 \theta = \frac{16}{25}$

Hence $\frac{1 - \cos^2 \theta}{1 + \cos^2 \theta} = \frac{1 - \frac{16}{25}}{1 + \frac{16}{25}} = \frac{9}{41}$

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If $\tan \theta = \sqrt{3}$, find the value of $\left(\frac{2 \sec \theta}{1 + \tan^2 \theta}\right)$

□□□□ $\sec^2 \theta = 1 + 3 = 4$

$\therefore \sec \theta = 2$

Hence $\frac{2 \sec \theta}{1 + \tan^2 \theta} = \frac{2 \times 2}{4} = 1$

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Read the following passage and answer the questions given at the end :

Students of Class XII presented a gift to their school in the form of an electric lamp in the shape of a glass hemispherical base surmounted by a metallic cylindrical top of same radius 21 cm and height 3.5 cm. The top was silver coated and the glass surface was painted red.

(i) What is the cost of silver coating the top at the rate of ₹ 5 per 100 cm² ?

(ii) What is the surface area of glass to be painted red ?

□□□□ (i) Surface Area of the top = $2 \times \frac{22}{7} \times 21 \times 3.5 = 462 \text{ cm}^2$

Cost of silver coating = $462 \times \frac{5}{100} = \text{Rs. } 23.10$

(ii) Surface Area of glass = $2 \times \frac{22}{7} \times 21 \times 21 = 2772 \text{ cm}^2$

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Find the probability that a leap year selected at random will contain 53 Sundays and 53 Mondays.

□□□□ 366 days = 52 weeks + 2 days

Total possible outcomes are 7 (SM, MT, TW, WTh, ThF, FS, SS)

Prob (having 53 Sundays & 53 Mondays) = $\frac{1}{7}$

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Find the value of p, if the mean of the following distribution is 7.5.

Classes	2-4	4-6	6-8	8-10	10-12	12-14
Frequency (fi)	6	8	15	p	8	4

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2-4	6	3	18
4-6	8	5	40
6-8	15	7	105
8-10	p	9	9p
10-12	8	11	88
12-14	4	13	52
	41 + p		303 + 9p

Mean = 7.5 = $\frac{303+9p}{41+p} \Rightarrow p = 3$

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□□□ Find a, b and c if it is given that the numbers a, 7, b, 23, c are in AP.

□□□□ a, 7, b, 23, c are in A.P

Let d be the common difference of AP.

$$\therefore a + d = 7 \quad \dots (i)$$

$$a + 3d = 23 \quad \dots (ii)$$

Solving (i) & (ii), $d = 8$

$$\therefore a = -1, b = 15, c = 31$$

□□

If m times the m^{th} term of an AP is equal to n times its n^{th} term, show that the $(m + n)^{\text{th}}$ term of the AP is zero.

□□□□ Given $m[a + (m - 1)d] = n[a + (n - 1)d]$

$$\Rightarrow a(m - n) + d(m^2 - m - n^2 + n) = 0$$

$$\Rightarrow (m - n)[a + (m + n - 1)d] = 0$$

$$\therefore m \neq n \Rightarrow a + (m + n - 1)d = 0$$

$$\Rightarrow a_{m+n} = 0$$

□□□ Find the values of k, for which the quadratic equation

$(k + 4)x^2 + (k + 1)x + 1 = 0$ has equal roots.

□□□□ For equal roots $(k + 1)^2 - 4(k + 4) \times 1 = 0$

$$\Rightarrow k^2 - 2k - 15 = 0$$

$$\Rightarrow (k + 3)(k - 5) = 0$$

$$\Rightarrow k = -3, 5$$

□□□ On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$ respectively. Find $g(x)$.

□□□□ $x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$

$$\Rightarrow (x - 2)g(x) = x^3 - 3x^2 + 3x - 2$$

$$\Rightarrow g(x) = \frac{(x - 2)(x^2 - x + 1)}{(x - 2)}$$

$$= x^2 - x + 1$$

□□

If the sum of the squares of zeros of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, find the value of k.

□□□□ Let the zeroes of polynomial $f(x)$ be α and β .

$$\therefore \alpha + \beta = 8 \text{ and } \alpha\beta = k$$

$$\therefore \alpha^2 + \beta^2 = 40$$

$$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 40$$

$$\Rightarrow 64 - 2k = 40$$

$$\Rightarrow k = 12$$

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In what ratio does the point P(-4, y) divide the line segment joining the points A(-6, 10) and B(3, -8) if it lies on AB. Hence find the value of y.

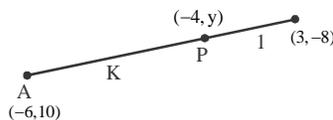
□□□□ Let AP : PB = k : 1

$$\therefore -4 = \frac{3k - 6}{k + 1}$$

$$\Rightarrow k = \frac{2}{7}$$

$$\therefore AP : PB = 2 : 7$$

$$\text{Hence } y = \frac{-8k + 10}{k + 1} = \frac{-8 \times \frac{2}{7} + 10}{\frac{2}{7} + 1} = 6$$



3□□

Prove that, a tangent to a circle is perpendicular to the radius through the point of contact.

□□□□ Given, To prove, figure

Correct proof

□□

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

$$\left. \begin{array}{l} \square\square\square\square \angle PAO = 90^\circ \text{ (radius } \perp \text{ tangent)} \\ \angle PBO = 90^\circ \end{array} \right\}$$

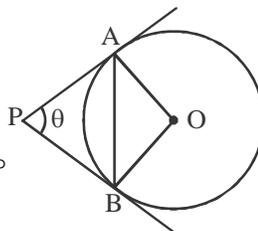
Now

$$\angle PAO + \angle AOB + \angle OBP + \angle BPA = 360^\circ$$

$$\Rightarrow 90^\circ + \angle AOB + 90^\circ + \angle BPA = 360^\circ$$

$$\Rightarrow \angle AOB + \angle BPA = 180^\circ$$

or $\angle AOB$ and $\angle BPA$ are supplementary.



3□□

In a right triangle, prove that the square of the hypotenuse is equal to the sum of squares of the other two sides.

□□□□ Correct given, To prove & figure

Correct proof

33□

If $\sin \theta + \cos \theta = p$ and $\sec \theta + \operatorname{cosec} \theta = q$, show that $q(p^2 - 1) = 2p$.

3

If the angle of elevation of a cloud from a point 10 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60° , find the height of the cloud from the surface of lake.

Let C represents the position of cloud and C' represents its reflection in the lake.

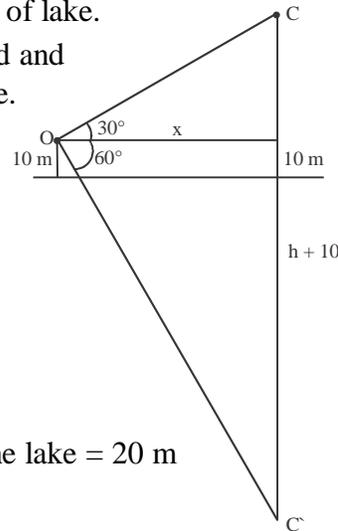
$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow x = h\sqrt{3} \quad \dots (i)$$

$$\tan 60^\circ = \sqrt{3} = \frac{h+20}{x} \quad \dots (ii)$$

Solving (i) and (ii) $h = 10$

\therefore Height of cloud from surface of the lake = 20 m



A vertical tower of height 20 m stands on a horizontal plane and is surmounted by a vertical flag-staff of height h. At a point on the plane, the angle of elevation of the bottom and top of the flag staff are 45° and 60° respectively. Find the value of h.

Let AC be the tower and CD be the flag-staff.

$$\tan 45^\circ = 1 = \frac{AC}{AB}$$

$$\Rightarrow AC = AB \quad \dots (i)$$

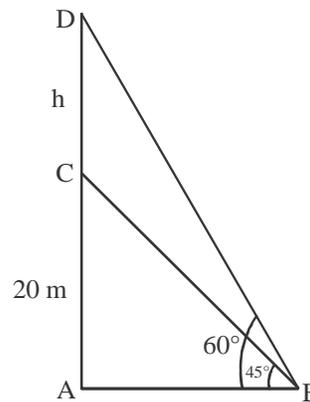
$$\tan 60^\circ = \sqrt{3} = \frac{AC + h}{AB}$$

$$\Rightarrow \sqrt{3} AB = AC + h \quad \dots (ii)$$

Using (i) and (ii)

$$AC(\sqrt{3} - 1) = h$$

$$\Rightarrow h = 20(\sqrt{3} - 1) \text{ m}$$



3

A solid iron cuboidal block of dimensions $4.4 \text{ m} \times 2.6 \text{ m} \times 1 \text{ m}$ is cast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

Internal radius of cylinder (r_2) = 30 cm = 0.30 m

Outer radius of cylinder (r_1) = $30 + 5 = 35 \text{ cm} = 0.35 \text{ m}$

Therefore $4.4 \times 2.6 \times 1 = \pi \times h \times ((0.35)^2 - (0.30)^2)$

$$= \pi \times h \times \frac{1}{100 \times 100} \times 65 \times 5$$

$$\Rightarrow h = \frac{352}{\pi} \text{ m or } 112 \text{ m}$$

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For the following frequency distribution, draw a cumulative frequency curve of 'more than' type and hence obtain the median value.

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

□□□□ Plotting points (0, 100) (10, 95) (20, 80) (30, 60) (40, 37) (50, 20) (60, 9)

and joining them.

Median = 34.3 (approx)

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$$\square\square\square\square \text{ (i) Surface Area of the top} = 2 \times \frac{22}{7} \times 21 \times 3.5 = 462 \text{ cm}^2$$

$$\text{Cost of silver coating} = 462 \times \frac{5}{100} = \text{Rs. } 23.10$$

$$\begin{aligned} \text{(ii) Surface Area of glass} &= 2 \times \frac{22}{7} \times 21 \times 21 \\ &= 2772 \text{ cm}^2 \end{aligned}$$

$$\square\square\square \text{ If } \tan \theta = \frac{3}{4}, \text{ find the value of } \left(\frac{1 - \cos^2 \theta}{1 + \cos^2 \theta} \right)$$

$$\square\square\square \sec^2 \theta = 1 + \frac{9}{16} = \frac{25}{16}$$

$$\therefore \cos^2 \theta = \frac{16}{25}$$

$$\text{Hence } \frac{1 - \cos^2 \theta}{1 + \cos^2 \theta} = \frac{1 - \frac{16}{25}}{1 + \frac{16}{25}} = \frac{9}{41}$$

$$\text{If } \tan \theta = \sqrt{3}, \text{ find the value of } \left(\frac{2 \sec \theta}{1 + \tan^2 \theta} \right)$$

$$\square\square\square \sec^2 \theta = 1 + 3 = 4$$

$$\therefore \sec \theta = 2$$

$$\text{Hence } \frac{2 \sec \theta}{1 + \tan^2 \theta} = \frac{2 \times 2}{4} = 1$$

$\square 3 \square$ Find the 11th term from the last term (towards the first term) of the AP 12, 8, 4, ..., -84.

$$\square\square\square \ l = -84$$

$$d = -4$$

$$t_{11} \text{ (from the end)} = -84 + 40 = -44$$

Solve the equation : $1 + 5 + 9 + 13 + \dots + x = 1326$

$$\square\square\square \frac{n}{2}(1 + x) = 1326 \quad \dots \text{ (i)}$$

$$x = 1 + (n - 1) \times 4 \quad \dots \text{ (ii)}$$

Solving (i) and (ii) $x = 101$

□□□

Find the value of p, if the mean of the following distribution is 7.5.

Classes	2-4	4-6	6-8	8-10	10-12	12-14
Frequency (fi)	6	8	15	p	8	4

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Class	Frequency (f)	x	fx
2-4	6	3	18
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8-10	p	9	9p
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12-14	4	13	52
41 + p			303 + 9p

$$\text{Mean} = 7.5 = \frac{303 + 9p}{41 + p} \Rightarrow p = 3$$

□□□

In a family of 3 children, find the probability of having at least one boy.

□□□□ Total number of outcomes = 8

Number of Favourable outcomes = 7

$$\text{Probability (having at least one boy)} = \frac{7}{8}$$

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In Fig. 4, PA is a tangent from an external point P to a circle with centre O. If $\angle POB = 115^\circ$, find $\angle APO$.

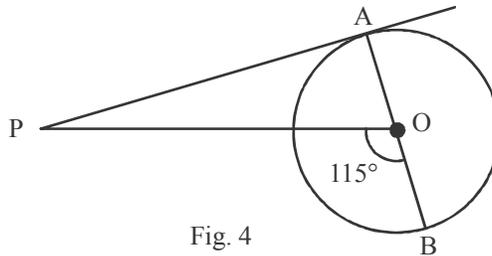


Fig. 4

$$\square\square\square\square \angle POA = 180^\circ - 115^\circ = 65^\circ$$

$\therefore OA \perp AP$

$$\text{therefore } \angle APO = 90^\circ - 65^\circ = 25^\circ$$

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500 persons are taking dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04 m^3 ?

□□□□ Let the rise in the water level be h

$$\therefore 500 \times .04 = 80 \times 50 \times h$$

$$\Rightarrow h = \frac{500 \times .04}{80 \times 50} = .005 \text{ m}$$

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If $\sin \theta + \cos \theta = p$ and $\sec \theta + \operatorname{cosec} \theta = q$, show that $q(p^2 - 1) = 2p$.

□□□□ LHS = $q(p^2 - 1) = (\sec \theta + \operatorname{cosec} \theta) ((\sin \theta + \cos \theta)^2 - 1)$

$$= \frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta} \times 2 \sin \theta \cos \theta$$

$$= 2 (\sin \theta + \cos \theta)$$

$$= 2p = \text{RHS}$$

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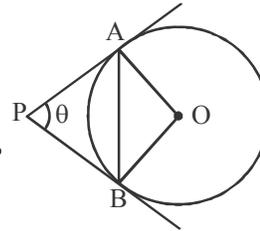
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On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$ respectively. Find $g(x)$.

□□□□ $x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$

$$\Rightarrow (x - 2) g(x) = x^3 - 3x^2 + 3x - 2$$

$$\Rightarrow g(x) = \frac{(x - 2)(x^2 - x + 1)}{(x - 2)}$$

$$= x^2 - x + 1$$

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If the sum of the squares of zeros of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, find the value of k .

□□□□ Let the zeroes of polynomial $f(x)$ be α and β .

$$\therefore \alpha + \beta = 8 \text{ and } \alpha\beta = k$$

$$\therefore \alpha^2 + \beta^2 = 40$$

$$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 40$$

$$\Rightarrow 64 - 2k = 40$$

$$\Rightarrow k = 12$$

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□□□□ Plotting points (0, 100) (10, 95) (20, 80) (30, 60) (40, 37) (50, 20), (60, 9)

and joining them.

Median = 34.3 (approx)

3□□

A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it

becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

□□□□ Let the fraction be $\frac{x}{y}$, $y \neq 0$

$$\text{Here } \frac{x-1}{y} = \frac{1}{3}$$

$$\text{and } \frac{x}{y+8} = \frac{1}{4}$$

$$\Rightarrow 3x - y = 3 \dots(i)$$

$$\text{and } 4x - y = 8 \dots(ii)$$

Solving (i) and (ii) $x = 5$, $y = 12$

\therefore Fraction is $\frac{5}{12}$

□□□

From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of same height and same diameter is hollowed out.

Find the total surface area of the remaining solid. $\left[\text{Use } \pi = \frac{22}{7} \right]$

□□□□ Radius = 0.7 cm

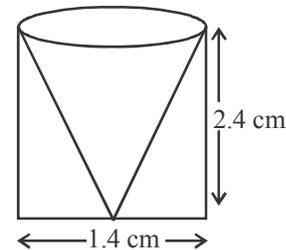
$$\text{Total Surface Area} = 2\pi rh + \pi r^2 + \pi r l$$

Here $r = 0.7$ cm, $h = 2.4$ cm

$$\therefore l = \sqrt{.49 + 5.76} = 2.5 \text{ cm}$$

$$\text{TSA} = \frac{22}{7} [2 \times .7 \times 2.4 + .49 + 0.7 \times 2.5]$$

$$= 17.6 \text{ cm}^2$$



□□□□

Class	Frequency (f)	x	fx
2-4	6	3	18
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6-8	15	7	105
8-10	p	9	9p
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12-14	4	13	52
		41 + p	303 + 9p

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$$\text{Mean} = 7.5 = \frac{303+9p}{41+p} \Rightarrow p = 3$$

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Read the following passage and answer the questions given at the end :

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(i) What is the cost of silver coating the top at the rate of ₹ 5 per 100 cm² ?

(ii) What is the surface area of glass to be painted red ?

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$$\text{Cost of silver coating} = 462 \times \frac{5}{100} = \text{Rs. } 23.10$$

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Find the 11th term from the last term (towards the first term) of the AP 12, 8, 4, ..., -84.

$$\square\square\square\square \quad l = -84$$

$$d = -4$$

$$t_{11} \text{ (from the end)} = -84 + 40 = -44$$

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Solve the equation : 1 + 5 + 9 + 13 + ... + x = 1326

$$\square\square\square\square \quad \frac{n}{2}(1+x) = 1326 \quad \dots \text{ (i)}$$

□□□□

$$x = 1 + (n-1) \times 4 \quad \dots \text{ (ii)}$$

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Solving (i) and (ii) x = 101

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If $\tan \theta = \frac{3}{4}$, find the value of $\left(\frac{1 - \cos^2 \theta}{1 + \cos^2 \theta} \right)$

$$\square\square\square\square \quad \sec^2 \theta = 1 + \frac{9}{16} = \frac{25}{16}$$

$$\therefore \cos^2 \theta = \frac{16}{25}$$

$$\text{Hence } \frac{1 - \cos^2 \theta}{1 + \cos^2 \theta} = \frac{1 - \frac{16}{25}}{1 + \frac{16}{25}} = \frac{9}{41}$$

□ □

If $\tan \theta = \sqrt{3}$, find the value of $\left(\frac{2 \sec \theta}{1 + \tan^2 \theta} \right)$

$$\square \square \square \sec^2 \theta = 1 + 3 = 4$$

$$\therefore \sec \theta = 2$$

$$\text{Hence } \frac{2 \sec \theta}{1 + \tan^2 \theta} = \frac{2 \times 2}{4} = 1$$

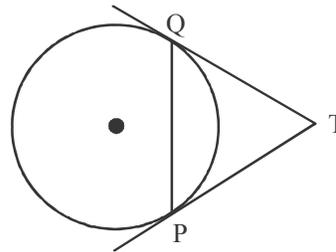
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Prove that the tangents at the extremities of any chord of a circle make equal angles with the chord.

□ □ □ Here $TQ = TP$

$\therefore \Delta TQP$ is isosceles

Hence $\angle TQP = \angle TPQ$



□ □ □

Two dice are thrown together once. Find the probability of getting a sum of more than 9.

□ □ □ Total number of outcomes = 36

Favourable outcomes are (5, 5), (4, 6), (6, 4), (6, 5), (5, 6), (6, 6)
i.e. 6 outcomes.

$$\text{Prob. (sum > 9)} = \frac{6}{36} = \frac{1}{6}$$

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Q. Nos. □ □ to 3 □ carry 3 marks each.

□ □ □

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□ □ □ Let the rise in the water level be h

$$\therefore 500 \times .04 = 80 \times 50 \times h$$

$$\Rightarrow h = \frac{500 \times .04}{80 \times 50}$$

$$= .005 \text{ m}$$

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If $\sin \theta + \cos \theta = p$ and $\sec \theta + \operatorname{cosec} \theta = q$, show that $q(p^2 - 1) = 2p$.

□□□□ LHS = $q(p^2 - 1) = (\sec \theta + \operatorname{cosec} \theta) ((\sin \theta + \cos \theta)^2 - 1)$

$$= \frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta} \times 2 \sin \theta \cos \theta$$

$$= 2 (\sin \theta + \cos \theta)$$

$$= 2p = \text{RHS}$$

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Prove that, a tangent to a circle is perpendicular to the radius through the point of contact.

□□□□ Given, To prove, figure

Correct proof

□□

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

□□□□ $\left. \begin{aligned} \angle PAO &= 90^\circ \text{ (radius } \perp \text{ tangent)} \\ \angle PBO &= 90^\circ \end{aligned} \right\}$

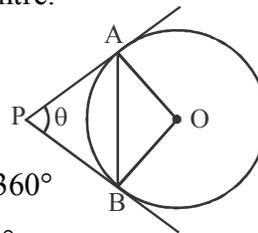
Now

$$\angle PAO + \angle AOB + \angle OBP + \angle BPA = 360^\circ$$

$$\Rightarrow 90^\circ + \angle AOB + 90^\circ + \angle BPA = 360^\circ$$

$$\Rightarrow \angle AOB + \angle BPA = 180^\circ$$

or $\angle AOB$ and $\angle BPA$ are supplementary.



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On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$ respectively. Find $g(x)$.

□□□□ $x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$

$$\Rightarrow (x - 2) g(x) = x^3 - 3x^2 + 3x - 2$$

$$\Rightarrow g(x) = \frac{(x - 2)(x^2 - x + 1)}{(x - 2)}$$

$$= x^2 - x + 1$$

□□

If the sum of the squares of zeros of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, find the value of k .

□□□□ Let the zeroes of polynomial $f(x)$ be α and β .

$$\therefore \alpha + \beta = 8 \text{ and } \alpha\beta = k$$

$$\therefore \alpha^2 + \beta^2 = 40$$

$$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 40$$

$$\Rightarrow 64 - 2k = 40$$

$$\Rightarrow k = 12$$

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3 Find a, b and c if it is given that the numbers a, 7, b, 23, c are in AP.

a, 7, b, 23, c are in A.P

Let d be the common difference of AP.

$$\therefore a + d = 7 \quad \dots (i)$$

$$a + 3d = 23 \quad \dots (ii)$$

Solving (i) & (ii) $d = 8$

$$\Rightarrow a = -1, b = 15, c = 31$$

If m times the m^{th} term of an AP is equal to n times its n^{th} term, show that the $(m + n)^{\text{th}}$ term of the AP is zero.

Given $m[a + (m - 1)d] = n[a + (n - 1)d]$

$$\Rightarrow a(m - n) + d(m^2 - m - n^2 + n) = 0$$

$$\Rightarrow (m - n)[a + (m + n - 1)d] = 0$$

$$\therefore m \neq n \Rightarrow a + (m + n - 1)d = 0$$

$$\Rightarrow a_{m+n} = 0$$

3 Find the values of k for which the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k - 1, 5k) are collinear.

Points A, B, C are collinear

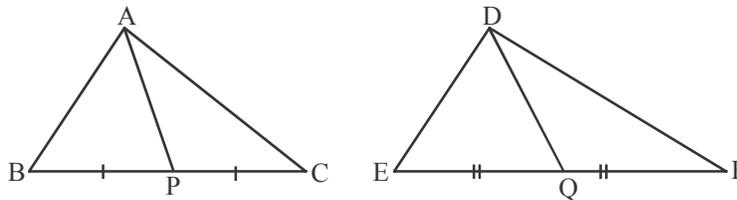
$$\Rightarrow (k + 1)(2k + 3 - 5k) + 3k(5k - 2k) + (5k - 1)(2k - 2k - 3) = 0$$

$$\Rightarrow 6k^2 - 15k + 6 = 0$$

$$\Rightarrow (k - 2)(2k - 1) = 0$$

$$\Rightarrow k = 2, \frac{1}{2}$$

33 Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding medians.



Here $\Delta ABC \sim \Delta DEF$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{2BP}{2EQ} = \frac{BP}{EQ} \text{ \& } \angle B = \angle E$$

$$\therefore \Delta ABP \sim \Delta DEQ$$

$$\Rightarrow \frac{AB}{DE} = \frac{AP}{DQ}$$

$$\Rightarrow \frac{AB^2}{DE^2} = \frac{AP^2}{DQ^2}$$

$$\Rightarrow \frac{\text{Ar}(\Delta ABC)}{\text{Ar}(\Delta DEF)} = \frac{AP^2}{DQ^2} \quad (\because \Delta ABC \sim \Delta DEF)$$

3

Find the value of k for which the quadratic equation $kx^2 + 1 - 2(k-1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation.

Equation can be written as

$$(k+1)x^2 - 2(k-1)x + 1 = 0$$

For equal roots $4(k-1)^2 - 4(k+1) = 0$

$$\Rightarrow k^2 - 3k = 0$$

$$\Rightarrow k(k-3) = 0$$

$$\Rightarrow k = 0, 3$$

For k = 0, equation is $x^2 + 2x + 1 = 0$

$$\Rightarrow x = -1, -1$$

For k = 3, equation is $4x^2 - 4x + 1 = 0$

$$\Rightarrow x = \frac{1}{2}, \frac{1}{2}$$

3

3

In an equilateral triangle ABC, D is a point on the side BC such that

$$BD = \frac{1}{3} BC. \text{ Prove that } 9AD^2 = 7AB^2.$$

Draw $AE \perp BC$

$\therefore \Delta ABC$ is an equilateral Δ

$$\therefore BE = \frac{BC}{2}$$

Now, $AD^2 = AE^2 + DE^2$ and $AB^2 = AE^2 + BE^2$

$$\Rightarrow AB^2 = AD^2 - DE^2 + BE^2$$

$$= AD^2 + (BE + DE)(BE - DE)$$

$$= AD^2 + \frac{BC}{3} \times \left(\frac{BC}{2} + \frac{BC}{2} - \frac{BC}{3} \right)$$

$$= AD^2 + \frac{2}{9} BC^2 = AD^2 + \frac{2}{9} AB^2$$

$$\Rightarrow 7AB^2 = 9AD^2$$

Prove that the sum of squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

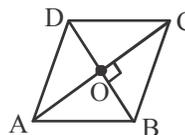
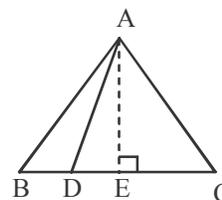
$AB^2 + BC^2 + CD^2 + AD^2$

$$= 4AB^2 (\because ABCD \text{ is a rhombus})$$

$$= 4(OA^2 + OB^2)$$

$$= 4 \left(\frac{AC^2}{4} + \frac{BD^2}{4} \right)$$

$$= AC^2 + BD^2$$



3

If the angle of elevation of a cloud from a point 10 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60° , find the height of the cloud from the surface of lake.

Let C represents the position of cloud and C' represents its reflection in the lake.

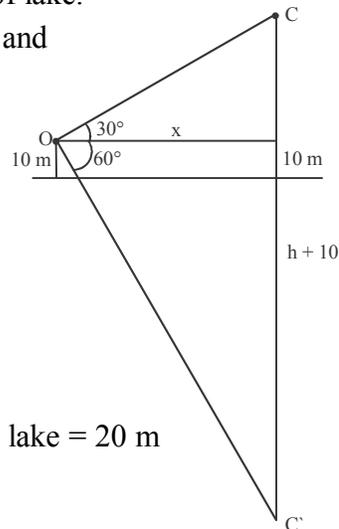
$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow x = h\sqrt{3} \quad \dots (i)$$

$$\tan 60^\circ = \sqrt{3} = \frac{h+20}{x} \quad \dots (ii)$$

Solving (i) and (ii) $h = 10$

\therefore Height of cloud from surface of the lake = 20 m



A vertical tower of height 20 m stands on a horizontal plane and is surmounted by a vertical flag-staff of height h. At a point on the plane, the angle of elevation of the bottom and top of the flag staff are 45° and 60° respectively. Find the value of h.

Let AC be the tower and CD be the flag-staff.

$$\tan 45^\circ = 1 = \frac{AC}{AB}$$

$$\Rightarrow AC = AB \quad \dots (i)$$

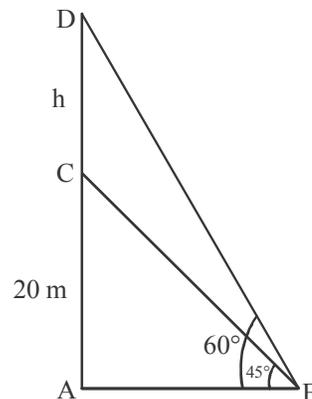
$$\tan 60^\circ = \sqrt{3} = \frac{AC + h}{AB}$$

$$\Rightarrow \sqrt{3} AB = AC + h \quad \dots (ii)$$

Using (i) and (ii)

$$AC(\sqrt{3} - 1) = h$$

$$\Rightarrow h = 20(\sqrt{3} - 1) \text{ m}$$



3

Show that $(12)^n$ cannot end with digit 0 or 5 for any natural number n.

$12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$

Since there is no factor of the form 5^m therefore 12^n can not end with digit 0 or 5 for any natural number n.

Prove that $(\sqrt{2} + \sqrt{5})$ is irrational.

Let us assume $\sqrt{2} + \sqrt{5}$ is rational number

Let $\sqrt{2} + \sqrt{5} = m$ where m is rational

$$\Rightarrow (\sqrt{2} + \sqrt{5})^2 = m^2$$

$$\Rightarrow m^2 = 7 + 2\sqrt{10}$$

$$\Rightarrow \sqrt{10} = \frac{m^2 - 7}{2}$$

$\therefore m$ is rational

$\therefore \frac{m^2 - 7}{2}$ is also rational

but $\sqrt{10}$ is irrational

$\Rightarrow \text{LHS} \neq \text{RHS}$

It means our assumption was wrong.

Hence $\sqrt{2} + \sqrt{5}$ is an irrational number.

- 3 For the following frequency distribution, draw a cumulative frequency curve of 'more than' type and hence obtain the median value.

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

Plotting points (0, 100) (10, 95) (20, 80) (30, 60) (40, 37) (50, 20) (60, 9)

and joining them.

Median = 34.3 (approx)

- 3 If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

Let the fraction be $\frac{x}{y}$, $y \neq 0$.

Here $\frac{x+1}{y-1} = 1$.

and $\frac{x}{y+1} = \frac{1}{2}$.

$\Rightarrow 2x - y = 1 \dots$ (i)

and $x - y = -2 \dots$ (ii)

Solving (i) & (ii)

$x = 3$, $y = 5$

\therefore fraction is $\frac{3}{5}$

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A hemispherical depression is cut out from one face of a cuboidal block of side 7 cm such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of the remaining solid.

□□□□ Here $r = \frac{7}{2}$ cm

$$\begin{aligned}\text{Total Surface Area} &= (5 \times 7^2) + \left(7^2 - \frac{49}{4}\pi\right) + 2 \times \pi \frac{49}{4} \\ &= \left(245 + 49 + \frac{49}{4}\pi\right) \text{cm}^2 \\ &= \left(294 + 49 + \frac{49}{4}\pi\right) \text{cm}^2 \\ &= 332.5 \text{ cm}^2 \text{ (approx)}\end{aligned}$$

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